S.NO: 22N1-UM **Course Code: MUA1**

A.D.M.COLLEGE FOR WOMEN, NAGAPATTINAM

(AUTONOMOUS)

B. Sc. (Chemistry/ Physics/ Geology) Degree Examination

I Semester – November 2022

AC I – MATHEMATICS I -

ALGEBRA, ANALYTICAL GEOMETRY OF 3D AND TRIGONOMETRY

Time: 3 hours **Maximum Marks: 75**

Section –A **10X2=20**

Answer **ALL** the Questions:

1. Find the 5th power of 11 using binomial theorem.
2. Find the coefficient of x^n in the series $1+\frac{b+ax}{1!}+\frac{(b+ax)^2}{2!}+.............+\frac{(b+ax)^n}{n!}$.
3. Find the Characteristic Polynomial of the Matrix $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.
4. State Calyley - Hamilton Theorem.
5. Prove that the lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-1}{-2}$ and $\frac{x-2}{2}=\frac{y-3}{-1}=\frac{z+4}{3}$ are coplanar.
6. Find the equation of a sphere which passes through the point (1, -2, 3) and the circle $z=0,\ x^2+y^2+z^2-9=0$.
7. Prove that $\sin^5\theta=\left(\frac{1}{2^4}\right)(\sin 5\theta-5\sin 3\theta+10\sin\theta)$.
8. Find the value of $\sin 3^\circ$ correct to three decimal places.
9. Show that $\sinh 2x = 2\sinh x\cosh x$.

10. Write the relational equations between hyperbolic and circular trigonometry functions of sine and cosine.

Section –B **5X5=25**

Answer **ALL** the Questions:

11. a) If a_r be the coefficient of x^r in the expansion of e^{e^x}, then show that

$$a_r = \frac{1}{r!}\left(\frac{1^r}{1!}+\frac{2^r}{2!}+\frac{3^r}{3!}+........\right).$$ Hence deduce that $\frac{1^3}{1!}+\frac{2^3}{2!}+\frac{3^3}{3!}+........=5e$.

(or)

b) Obtain the expansion of $\log\left(1+\frac{1}{n}\right)$ in ascending powers of $\frac{1}{2n+1}$ and show that $\log\left(1+\frac{1}{n}\right)$ lies between $\frac{2}{2n+1}$ and $\frac{2n+1}{2n(n+1)}$ where n is positive.

12. a) Verify that A = $\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ satisfies its own characteristic equation and hence find A^4.

(or)

b) Using Cayley - Hamilton theorem find A^{-1} where A = $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

13. a) Find the shortest distance and equation of the line of shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

(or)

b) Find the equation of the sphere through the circle $x^2+y^2+z^2+2x+3y+6=0;\ x-2y+4z-9=0$ and the centre of the sphere $x^2+y^2+z^2-2x+4y-6z+5=0$.

14. a) Expand $\cos^4\theta\sin^3\theta$ in a series of sines of multiples of θ.

(or)

b) Solve approximately $\sin\left(\frac{\pi}{6}+\theta\right)=0.51$

15. a) Prove that $\cosh u=\sec\theta$ if $u=\log_e\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

(or)

b) Separate into real and imaginary parts of $\tan^{-1}(x+iy)$.

Section –C **3 X 10 = 30**

Answer any **THREE** Questions:

16. If n is a positive integer and $\frac{(1+x)^n}{(1-x)^4}=a_0+a_1x+a_2x^2+......+a_nx^n+...,$, show that $a_0+a_1+a_2+......+a_{n-1}=\frac{1}{3}n(n+2)(n+7)2^{n-4}$.

17. Find all the Eigen values and Eigenvectors of the matrix $\begin{pmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{pmatrix}$.

18. Find the equation of the sphere through the points (0, 0, 0), (0, 1, -1), (-1, 2, 0) and (1, 2, 3).

19. Expand $\sin 7\theta$ in powers of $\cos\theta$ and $\sin\theta$. Hence prove that $\frac{\sin 7\theta}{\sin\theta}=7-56\sin^2\theta+112\sin^4\theta-64\sin^6\theta$.

20. If $\tan(\theta+i\phi)=\cos\alpha+i\sin\alpha$, then prove that (i) $\theta=\frac{n\pi}{2}+\frac{\pi}{4}$ and (ii) $\phi=\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$.
